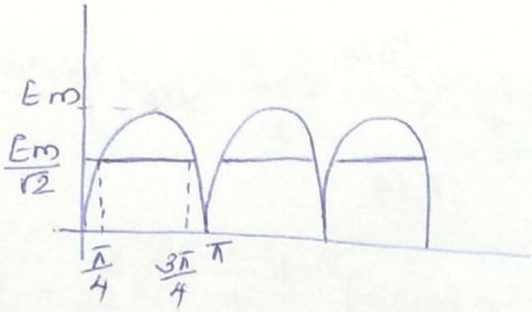
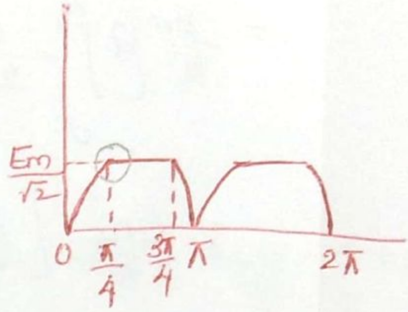
A full wave rectified sine wave is clipped at 0.707 of its max. value as shown. Find the avg. and rms values of the function.





$$E_m \sin \omega t = \frac{E_m}{\sqrt{2}}$$

$$\Rightarrow \sin \omega t = \frac{1}{\sqrt{2}} \Rightarrow \omega t = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$\omega t = \frac{\pi}{4}$$

Here the sine wave is clipped at the level $\frac{Em}{\sqrt{2}}$. so at the point it clips ~~the~~ (shown in circle) the value of the wave is $\frac{Em}{\sqrt{2}}$ ie $E_m \sin \omega t = \frac{E_m}{\sqrt{2}}$.

The eqn. of the wave is

$0 < t < \frac{\pi}{4}$; $e = E_m \sin \omega t$

$\frac{\pi}{4} < t < \frac{3\pi}{4}$; $e = \frac{E_m}{\sqrt{2}}$

$\frac{3\pi}{4} < t < \pi$; $e = E_m \sin \omega t$

$$E_{avg.} = \frac{1}{\pi}\left[\int_0^{\pi/4} E_m \sin \omega t \, d\omega t + \int_{\pi/4}^{3\pi/4} \frac{E_m}{\sqrt{2}} \, d\omega t + \int_{3\pi/4}^{\pi} E_m \sin \omega t \, d\omega t\right]$$

$$= \frac{E_m}{\pi}\left[\left(-\cos \omega t\right)_0^{\pi/4} + \left[\frac{1}{\sqrt{2}} \omega t\right]_{\pi/4}^{3\pi/4} + \left[-\cos \omega t\right]_{3\pi/4}^{\pi}\right]$$

$$= \frac{E_m}{\pi}\left[-\left(\frac{1}{\sqrt{2}} - 1\right) + \frac{1}{\sqrt{2}}\left(\frac{3\pi}{4} - \frac{\pi}{4}\right) + -\left(-1 + \frac{1}{\sqrt{2}}\right)\right]$$

$$= \frac{E_m}{\pi}\left[-\frac{1}{\sqrt{2}} + 1 + \frac{3\pi}{4\sqrt{2}} - \frac{\pi}{4\sqrt{2}} + 1 - \frac{1}{\sqrt{2}}\right]$$

$$= \frac{E_m}{\pi}\left[-\frac{2}{\sqrt{2}} + 2 + \frac{2\pi}{4\sqrt{2}}\right] = \frac{E_m}{\pi}\left[-1.414 + 2 + 1.11\right]$$

$$= \frac{E_m}{\pi}\left[1.696\right] = 0.539\, E_m$$

$$(E_{rms})^2 = \frac{1}{\pi}\left[\int_0^{\pi/4} E_m^2 \sin^2\omega t\, d\omega t + \int_{\pi/4}^{3\pi/4} \frac{E_m^2}{2}\, d\omega t + \int_{3\pi/4}^{\pi} E_m^2 \sin^2\omega t\, d\omega t\right]$$

$$= \frac{E_m^2}{\pi}\left[2\int_0^{\pi/4}\left(\frac{1-\cos 2\omega t}{2}\right) + \int_{\pi/4}^{3\pi/4} \frac{E_m^2}{2}\, d\omega t\right]$$

$$= \frac{E_m^2}{2\pi}\left[2\left(\omega t - \frac{\sin 2\omega t}{2}\right)_0^{\pi/4} + \left(3\frac{\pi}{4} - \frac{\pi}{4}\right)\right]$$

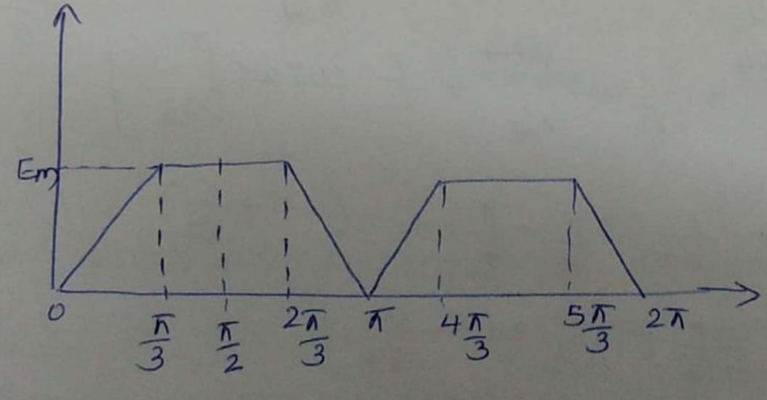
$$= \frac{E_m^2}{2\pi}\left[2\left(\frac{\pi}{4} - \frac{1}{2}\right) + \frac{2\pi}{4}\right]$$

$$= \frac{E_m^2}{2\pi}\left[\frac{\pi}{2} - 1 + \frac{\pi}{2}\right] = \frac{E_m^2}{2\pi}\left[\pi - 1\right]$$

$$= E_m^2 \times 0.34$$

$$E_{rms} = \sqrt{0.34 E_m^2} = \underline{\underline{0.584\, E_m}}$$

8. Find the avg. and rms value of the given trapezoidal wave form.



Here curve is symmetrical about $\pi/2$. so we have to consider 0 to $\frac{\pi}{2}$ only.
(Similar to Q. 3)

$$E_{avg.} = \frac{\text{area under the curve}}{\text{base}}$$

$$= \frac{1}{\pi/2}\left[\underbrace{\frac{1}{2}\times\frac{\pi}{3}\times E_m}_{\frac{1}{2}bh} + \underbrace{\left(\frac{\pi}{2}-\frac{\pi}{3}\right)E_m}_{l\times b}\right] = \frac{2E_m}{\pi}\left[\frac{\pi}{6}+\frac{\pi}{3}\right]$$

$$= \frac{2E_m}{\pi}\cdot\frac{\pi}{3} = \frac{2}{3}E_m$$

$$(E_{rms})^2 = \frac{1}{\pi/2}\left[\int_0^{\pi/3}\left(\frac{E_m}{(\pi/3)^2}\,\omega t\right)^2 d\omega t + \int_{\pi/3}^{\pi/2} E_m^2\, d\omega t\right]$$

$$= \frac{E_m^2}{\pi/2}\left[\int_0^{\pi/3}\frac{3^2}{\pi^2}(\omega t)^2\, d\omega t + \int_{\pi/3}^{\pi/2} d\omega t\right]$$

$$= \frac{2E_m^2}{\pi}\left[\frac{9}{\pi^2}\left[\frac{(\omega t)^3}{3}\right]_0^{\pi/3} + \left[\omega t\right]_{\pi/3}^{\pi/2}\right]$$

$$= \frac{2E_m^2}{\pi}\left[\frac{\cancel{9}}{\cancel{\pi^2}}\times\frac{1}{3}\times\frac{\cancel{\pi^3}}{\cancel{27}\,3} + \frac{\pi}{2} - \frac{\pi}{3}\right]$$

$$= \frac{2E_m^2}{\pi}\left[\frac{\pi}{9} + \frac{\pi}{6}\right] = \cancel{2}\frac{E_m^2}{\cancel{\pi}} \times \frac{5\cancel{\pi}}{\cancel{18}\,9}$$

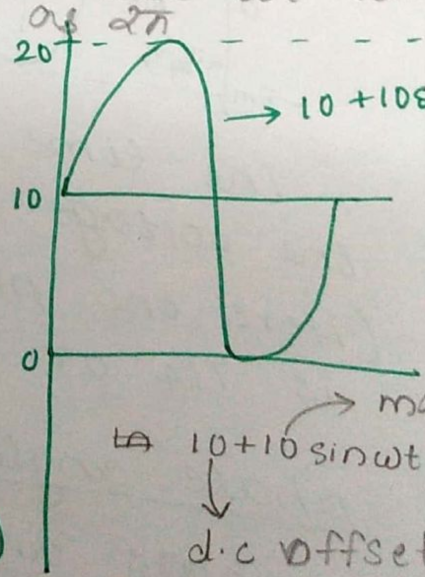
$$= \frac{5E_m^2}{9}$$

$$E_{rms} = \frac{\sqrt{5}\,E_m}{3}$$

9. Find the avg. and rms values of current.

$i(t) = 10 + 10\sin\omega t$ → In this case we have to take base as 2π

$i_{avg.} = 10\,A$ since ~~10 sin~~. the avg. value of $10\sin\omega t$ is zero in a complete cycle.



max. value

↳ 10+10 sin ωt,

d.c offset

$$i^2 = [10(1+\sin\omega t)]^2$$

$$= 100(1+\sin\omega t)^2$$

$$= 100(1+\sin^2\omega t + 2\sin\omega t)$$

$$= 100\left(1 + 2\sin\omega t + \frac{1-\cos 2\omega t}{2}\right)$$

$$= 100 + 200\sin\omega t + 50 - 50\cos 2\omega t$$

$$= 150 + 200\sin\omega t - 50\cos 2\omega t$$

$$(I_{rms})^2 = \frac{1}{2\pi}\int_0^{2\pi} i^2 \, d\omega t$$

$$= \frac{1}{2\pi}\int_0^{2\pi} 150 + 200\sin\omega t - 50\cos 2\omega t \, d\omega t$$

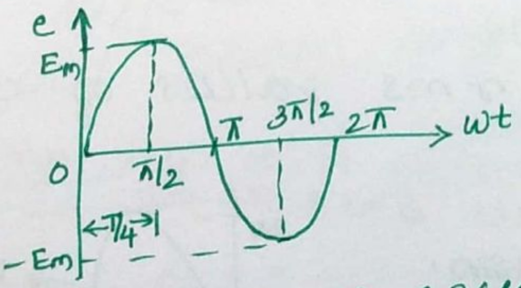
$$= \frac{1}{2\pi}\left[150\,\omega t - 200\cos\omega t - \frac{50}{2}\sin 2\omega t\right]_0^{2\pi}$$

$$= \frac{1}{2\pi}\left[150 \times 2\pi - 0 - 0\right] = \underline{150}$$

$$I_{rms} = \sqrt{150} = \underline{12.24\ A}$$

<u>phase</u>

phase of an alternating quantity is the fraction of the time period or cycle that has elapsed since it has last passed from the chosen zero position or origin.



The time is counted from the instant the voltage is zero and becoming positive. (Here the phase of max. positive value is T/4 or π/2 radians).

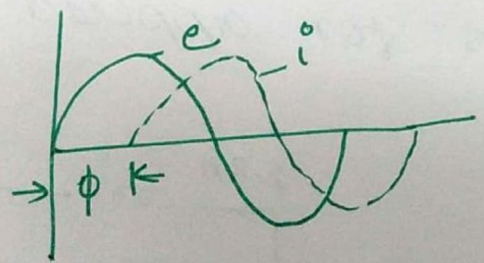
<u>phase angle</u>

phase angle φ is the equivalent of phase in radians or degree.
(The phase angle of the max. value of the above sinusoidal voltage is π/2 radians or 90 degree.)

phase difference

when two alternating quantities of same frequency have different zero position the are said to have a phase difference. phase difference between two alternating quantities is the fractional part of time period through which one alternating quantity has advanced over another alternating quantity. Two alternating quantities are in phase when both pass through their zero value and also attain their max. value at the same instant. Two alternating quantities are out of phase if they reach their min. and max. values at different times but always have an equal phase angle between them.

Lagging and leading quantities



Here e is the leading qty. & i is the lagging qty.

$e = E_m \sin \omega t$

$i = I_m (\sin \omega t - \phi)$

phasor representation of sinusoidal quantity.

Refer the slides given (a.c ckts.)